O LEVEL MATHEMATICS COMPLETE REVISION GUIDE

A Rigorous 12-Week Syllabus Breakdown, Topic Checklist, and Exam Strategies

Syllabus Code: 4024 / 0580 Complete Alignment
Core Domains: Algebra, Geometry, Trigonometry & Statistics

Prepared for Mock & Final Exam Excellence

LifeWithBooks Overview Series

About This Guide

The **O Level Mathematics Complete Revision Guide** is designed to support students building confidence before their mock and final examinations. Mathematics at the Ordinary Level requires not only a deep theoretical comprehension of core rules but also a highly disciplined approach to problem-solving, structured layout of mathematical arguments, and strategic agility across both non-calculator and calculator examination papers.

This guide serves as an extensive, comprehensive structural architecture to break down the vast syllabus into manageable, high-yielding review blocks. By presenting a definitive topic checklist across the four major foundational pillars—Algebra, Geometry, Trigonometry, and Statistics—and constructing a rigorous 12-week structured framework, this guide ensures no mathematical concept is left unreviewed.

Who It Is For

This publication is specifically curated for standard O Level Mathematics candidates, teachers looking for an objective-driven curriculum review layout, and private students seeking an intensive, self-paced revision timeline. It addresses common pain points such as numerical precision errors, mechanical misunderstandings in algebraic manipulation, proof structures in geometry, and data interpretation errors under timed conditions.

How to Use This Resource

Use this *LifeWithBooks* analytical overview to map out, plan, and audit your study timeline. It is essential to understand that this guide outlines the structural frameworks, foundational theorems, derivations, and step-by-step methodology needed for mastery. Students must obtain official past assessment papers and official learning materials from trusted publishers and examination bodies for complete practical preparation and timed testing.

Domain 1: O Level Mathematics Topic Checklist

Use this exhaustive structural checklist to track your revision progress. Ensure you can solve standard exam questions for each sub-topic without external assistance before ticking the completion status.

Section A: Algebraic Manipulation & Core Equations

Sub-Topic Component	Core Knowledge & Examination Skills Required	Status
Indices & Standard Form	Application of the prime laws of indices ($a^m \times a^n = a^{m+n}$, $(a^m)^n = a^{mn}$, $a^{-n} = 1/a^n$, $a^{1/n} = \sqrt[n]{a}$). Expressing numbers in scientific notation ($A \times 10^n$ where $1 \leq A < 10$). Operations in standard form without calculators.	[ ] Complete
Algebraic Expansion & Factorization	Expanding products of algebraic expressions including identities: $(a+b)^2 = a^2+2ab+b^2$, $(a-b)^2 = a^2-2ab+b^2$, and $a^2-b^2 = (a-b)(a+b)$. Factorization by grouping, quadratic factorization, and extraction of common terms.	[ ] Complete
Algebraic Fractions	Simplifying complex fractions via factorization. Addition, subtraction, multiplication, and division of fractions with variable denominators. Solving fractional algebraic equations reducible to linear or quadratic configurations.	[ ] Complete
Linear & Simultaneous Equations	Solving multi-step linear equations. Solving simultaneous linear equations in two unknowns using elimination, substitution, and graphical intersection techniques. Word problems mapping to simultaneous equations.	[ ] Complete
Quadratic Equations & Functions	Solving quadratic equations ($ax^2 + bx + c = 0$) via factorization, completing the square, and using the quadratic formula. Analyzing the nature of roots and sketching quadratic graphs highlighting turning points and intercepts.	[ ] Complete
Linear Inequalities	Solving first-degree inequalities in one variable. Representing solution sets on a real number line. Setting up and solving sets of simultaneous inequalities. Graphical representation of feasible regions in two dimensions.	[ ] Complete
Sequences & Series	Identifying patterns in linear, quadratic, and geometric configurations. Deriving the general expression for the n-th term (T_n). Problem-solving with multi-layered, non-standard visual patterns.	[ ] Complete

Section B: Geometric Principles & Transformations

Sub-Topic Component	Core Knowledge & Examination Skills Required	Status
Angle Properties & Polygons	Parallel line theorems (alternate, corresponding, interior angles). Interior and exterior angles of convex polygons. Interior angle sum: $(n-2) \times 180°$. Exterior angle sum: $360°$. Properties of regular polygons.	[ ] Complete
Circle Theorems	Angles at center and circumference; angle in a semi-circle; angles in same segment; cyclic quadrilateral properties (opposite angles sum to $180°$); tangent theorems (radius perpendicular to tangent, tangents from external point). Alternate segment theorem.	[ ] Complete
Congruence & Similarity	Proving triangle congruence via SSS, SAS, ASA, RHS. Scale factors for similar shapes. Area ratios ($A_1/A_2 = (l_1/l_2)^2$) and volume ratios ($V_1/V_2 = (l_1/l_2)^3$) for similar solids.	[ ] Complete
Mensuration & Perimeter/Area	Arc length ($s = r\theta$ or $(\theta/360)\times 2\pi r$) and sector area ($A = 0.5r^2\theta$ or $(\theta/360)\times \pi r^2$). Surface area and volume of prisms, cylinders, pyramids, cones, and spheres. Composite geometric structures.	[ ] Complete
Coordinate Geometry	Calculating gradient (m), midpoint, and length of a line segment given two coordinates. Equation of a straight line ($y = mx + c$). Parallel and perpendicular line relationships ($m_1 \times m_2 = -1$).	[ ] Complete
2D Transformations	Mapping coordinates and shapes using translation (vectors), reflection (mirror lines), rotation (center, angle, direction), and enlargement (center, scale factor including fractional/negative factors). Matrix transformations.	[ ] Complete

Section C: Trigonometry & Bearings

Sub-Topic Component	Core Knowledge & Examination Skills Required	Status
Right-Angled Trigonometry	Application of Pythagoras' Theorem ($a^2 + b^2 = c^2$). Trigonometric ratios: $\sin\theta$, $\cos\theta$, $\tan\theta$. Finding unknown side lengths and interior acute angles. Word problems involving angles of elevation and depression.	[ ] Complete
Non-Right Trigonometry	Area of any triangle ($A = 0.5ab \sin C$). The Sine Rule ($a/\sin A = b/\sin B = c/\sin C$). The Cosine Rule ($a^2 = b^2 + c^2 - 2bc \cos A$). Application to obtuse-angled scenarios and geometric modeling.	[ ] Complete
Bearings & Navigation	Three-figure notation system (e.g., $045°$, $210°$). Measuring angles clockwise from true North. Forward and back bearing relationships ($\theta \pm 180°$). Complex multi-point navigation word puzzles.	[ ] Complete
Three-Dimensional Problems	Identifying lines of greatest slope. Finding angles between a line and a plane. Application of Pythagoras and standard trigonometric ratios inside multi-planar 3D solids (pyramids, wedges, cuboids).	[ ] Complete

Section D: Statistics & Probability Theory

Sub-Topic Component	Core Knowledge & Examination Skills Required	Status
Data Representation	Construction and evaluation of bar charts, pie charts, pictograms, and line graphs. Frequency density calculation and histogram rendering for unequal class intervals. Extraction of raw values from graphical arrays.	[ ] Complete
Measures of Central Tendency	Calculation of Mean, Median, Mode, and Modal Class for discrete, grouped, and ungrouped data. Understanding the mathematical properties, advantages, and functional limitations of each individual measure.	[ ] Complete
Measures of Dispersion	Determining range, quartiles, and Interquartile Range (IQR). Constructing and interpreting cumulative frequency curves. Drawing and comparing box-and-whisker plots to analyze data distributions.	[ ] Complete
Elementary Probability	Probability scale ranging from 0 to 1. Calculating experimental and theoretical probabilities. Single event modeling and sample space diagrams. Mutually exclusive and complementary events.	[ ] Complete

Sub-Topic Component	Core Knowledge & Examination Skills Required	Status
Probability Trees & Venns	Constructing probability tree diagrams for independent and dependent (without replacement) sequential events. Conditional probability problems using formal notations and overlapping sets in Venn diagrams.	[ ] Complete

Comprehensive Theoretical Reference & Formulary

Before embarking on the 12-week intensive plan, a student must master the core theoretical machinery. Below is the explicit documentation of formulas, constraints, derivations, and common structural missteps.

1. The Algebraic Engine

Quadratic Solutions

For any standard quadratic expression configuration matching $ax^2 + bx + c = 0$ where $a \neq 0$, the variable paths are resolved via the quadratic formula:

$$x = -b \pm \sqrt{[b^2 - 4ac]} / (2a)$$

The value under the radical sign, $\Delta = b^2 - 4ac$, is the discriminant. If $\Delta > 0$, the function has two distinct real roots (crosses the x-axis twice). If $\Delta = 0$, there is exactly one repeated real root (the turning point is tangent to the x-axis). If $\Delta < 0$, no real roots exist.

> **NON-CALCULATOR OPERATIONAL MASTERY: COMPLETING THE SQUARE**
>
> When asked to express a quadratic expression in the perfect square form $a(x - h)^2 + k$ without a calculator, always isolate a, take half of the linear x coefficient, square it, and then add and subtract it within the structural parenthesis. This explicitly yields the minimum or maximum vertex point coordinates at (h, k).

Laws of Indices and Exponential Manipulation

Analytical arithmetic operations require seamless manipulation of indices. The global structural definitions are:

- Multiplication Rule: $x^a \times x^b = x^{a+b}$
- Division Rule: $x^a \div x^b = x^{a-b}$
- Power of a Power: $(x^a)^b = x^{ab}$
- Negative Index: $x^{-a} = 1 / x^a$
- Fractional Root Index: $x^{p/q} = \sqrt[q]{(x^p)} = (\sqrt[q]{x})^p$
- Zero Index Identity: $x^0 = 1$ (for $x \neq 0$)

2. Geometric Proofs, Mensuration, & Circle Properties

Properties of Circles

Circle geometry forms a substantial portion of Paper 1 and Paper 2. The standard theorems are categorized below:

Theorem Name	Geometric Property Description	Exam Notation Rule
Angle at Center	The angle subtended by an arc at the center is twice the angle subtended at any point on the remaining circumference.	"∠ at center is 2 × ∠ at circum."
Angles in Same Segment	Angles subtended by the same arc in the same segment of a circle are equal. This is often called the "bowtie" configuration.	"∠s in same segment are equal"
Cyclic Quadrilateral	The opposite interior angles of a four-sided shape inscribed inside a circle always sum to exactly $180°$.	"opp ∠s of cyclic quad sum to 180°"
Alternate Segment	The angle between a tangent and a chord through the point of contact is equal to the angle subtended by the chord in the alternate segment.	"∠ in alternate segment"

Example Breakdown: Working with Similar Solids

Problem: Two mathematically similar cylinders have heights of $40\ cm$ and $60\ cm$ respectively. If the total surface area of the smaller cylinder is $250\ cm^2$, find the surface area of the larger cylinder.

Step-by-step Solution:

1. Find the base linear scale factor (k): $k = h_2 / h_1 = 60 / 40 = 1.5$.
2. The area ratio is the square of the linear scale factor: $k^2 = (1.5)^2 = 2.25$.
3. Set up the calculation for the new area: $Area_{large} = 2.25 \times Area_{small}$.
4. Execute calculation: $Area_{large} = 2.25 \times 250 = 562.5\ cm^2$.

The 12-Week Structured Revision Syllabus

The following pages outline a comprehensive 12-week revision blueprint. Each week isolates a vital subset of the O Level syllabus, details core subtopics, provides theoretical insights, and supplies exam tips customized for non-calculator (Paper 1) and calculator (Paper 2) assessments.

Week 1 Supplement: Detailed Theoretical Analysis of Absolute Number Systems

In Ordinary Level Mathematics, a complete understanding of numbers forms the bedrock of all subsequent algebraic and geometric proofs. Students must distinguish clearly between various nested sets of numbers. The set of Natural Numbers (***&mathbb{N}***) comprises positive integers starting from 1. Integers (***&mathbb{Z}***) expand this to include zero and negative whole numbers.

Rational Numbers (***&mathbb{Q}***) are defined as numbers that can be written as a fraction p/q, where p and q are integers and $q \neq 0$. Irrational numbers cannot be expressed this way; their decimal expansions continue forever without repeating, such as π or $\sqrt{2}$.

Deep Dive: Prime Factor Decomposition

The Fundamental Theorem of Arithmetic states that every integer greater than 1 is either a prime number itself or can be represented as a unique product of prime numbers. This is known as prime factor decomposition. For example, the number 1200 can be methodically broken down using a factor tree or repeated division:

$$1200 = 2 \times 600 = 2^2 \times 300 = 2^3 \times 150 = 2^4 \times 75 = 2^4 \times 3 \times 25 = 2^4 \times 3^1 \times 5^2$$

This structured representation simplifies finding common factors and multiples for large sets of numbers.

Comprehensive Practice Examples for Week 1

Extended Example 1.1: Finding HCF and LCM via Index Powers

Given two numbers expressed as prime factors, $A = 2^3 \times 3^2 \times 5^1$ and $B = 2^2 \times 3^3 \times 7^1$, find the Highest Common Factor (HCF) and the Lowest Common Multiple (LCM).

Detailed Explanation:

To find the HCF, compare the powers of the common prime factors (2 and 3) and select the lowest exponent for each:

$HCF = 2^{min(3,2)} \times 3^{min(2,3)} = 2^2 \times 3^2 = 4 \times 9 = 36$.

To find the LCM, take the highest exponent of every prime factor present in either number:

$LCM = 2^{max(3,2)} \times 3^{max(2,3)} \times 5^{max(1,0)} \times 7^{max(0,1)} = 2^3 \times 3^3 \times 5^1 \times 7^1$

$LCM = 8 \times 27 \times 5 \times 7 = 7560$.

Week 1: Arithmetic Foundations, Primes, & Estimation

Syllabus Coverage & Core Concepts

This block focuses on rebuilding arithmetic confidence. Topics include prime factor decomposition, Lowest Common Multiple (LCM), Highest Common Factor (HCF), sets of real numbers (rational, irrational, integers, natural numbers), approximation, significant figures, decimal bounds, and order of operations.

Comprehensive Theoretical Explanations

Every composite integer can be uniquely expressed as a product of prime powers. This foundational property allows us to systematically find the HCF and LCM. To calculate the HCF of two numbers expressed as prime factors, take the *lowest* power of each common prime factor. To find the LCM, take the *highest* power of all prime factors present in either number.

Estimation and absolute error bounds are critical for validation. For a value rounded to a specific degree of precision, its true value (x) lies within upper and lower operational limits:

$$\text{Lower Bound} \leq \text{True Value} < \text{Upper Bound}$$

When dividing numbers to find maximum and minimum bounds, the absolute relationships are:

- Maximum Quotient = Maximum Dividend ÷ Minimum Divisor
- Minimum Quotient = Minimum Dividend ÷ Maximum Divisor

PAPER 1: NON-CALCULATOR STRATEGY

In estimations, round every input to exactly one significant figure before performing basic operations. For example, estimate $\sqrt{[42.3 \times 8.91]} / 0.198$ by evaluating $\sqrt{[40 \times 9]} / 0.2 = \sqrt{360} / 0.2 \approx 19 / 0.2 = 95$. This prevents complex arithmetic in early questions.

PAPER 2: CALCULATOR OPTIMIZATION

Avoid rounding intermediate values displayed on your screen. Keep full precision in the calculator's memory registers and apply rounding rules only at the final step to avoid compounding rounding errors.

Practice Problems & Step-by-Step Solutions

Problem 1: Bounds in Arithmetic Operations

A rectangular field has length $L = 80\ ext\{\ m\}$ and width $W = 35\ ext\{\ m\}$, both rounded to the nearest five meters. Calculate the exact upper bound for the total area of the field.

Solution:

Since the values are rounded to the nearest $5\ ext\{\ m\}$, the interval variation range is $\pm 2.5\ ext\{\ m\}$.

Upper Bound of Length (L_{UB}) = $80 + 2.5 = 82.5\ ext\{\ m\}$

Upper Bound of Width (W_{UB}) = $35 + 2.5 = 37.5\ ext\{\ m\}$

Maximum possible Area = $L_{UB} \times W_{UB} = 82.5 \times 37.5 = 3093.75\ ext\{\ m\}^2$

Problem 2: Prime Decomposition Analysis

Express 504 as a product of its prime factors. Hence, find the smallest integer k such that $504k$ is a perfect square.

Solution:

Perform repeated division by prime numbers: $504 = 2 \times 252 = 2 \times 2 \times 126 = 2 \times 2 \times 2 \times 63 = 2^3 \times 3^2 \times 7^1$.

For $504k$ to be a perfect square, the exponent of every prime factor in its prime factorization must be an even number.

To make the exponents of 2^3 and 7^1 even, we must multiply by one factor of 2 and one factor of 7.

Therefore, $k = 2 \times 7 = 14$.

Week 2 Supplement: Deep Dive into Quadratic Factorization Techniques

Mastering algebra requires moving beyond simple expansion to identifying underlying structural patterns. When dealing with a quadratic expression of the form $ax^2 + bx + c$, factorization involves finding two numbers that multiply to give ac and add up to give b. This process allows us to split the middle linear term and factorize the expression by grouping.

The Difference of Two Squares Identity

One of the most frequently tested algebraic identities is the difference of two squares. It states that an expression in the form $a^2 - b^2$ can always be factorized into two binomial factors:

$$a^2 - b^2 = (a - b)(a + b)$$

Recognizing this pattern is essential for simplifying complex algebraic fractions.

Comprehensive Practice Examples for Week 2

Extended Example 2.1: Factorization by Grouping and Identity Application

Factorize the expression completely: $6x^2 - 4am + 3mx - 8ax$.

Detailed Explanation:

First, rearrange the terms to group common variables together:

$= 6x^2 + 3mx - 8ax - 4am$

Factor out the greatest common factor from the first two terms, and then from the last two terms:

$= 3x(2x + m) - 4a(2x + m)$

Notice that $(2x + m)$ is now a common binomial factor. Factor it out to get the final result:

$= (2x + m)(3x - 4a)$.

Week 2: Algebraic Manipulation — Expansion, Factorization & Fractions

Syllabus Coverage & Core Concepts

This block focuses on core algebraic manipulation. Topics include expanding single and double sets of brackets, working with algebraic identities, factorizing quadratic expressions, grouping terms, and simplifying complex algebraic fractions.

Comprehensive Theoretical Explanations

Algebraic structures follow predictable rules. Expanding brackets involves distributing terms systematically across additions or subtractions, often using the FOIL method (First, Outer, Inner, Last). Factorization reverses this process by turning an expression back into a product of simpler factors.

The three core algebraic identities are essential tools for simplification:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a - b)(a + b)$$

When working with algebraic fractions, never cancel terms that are added or subtracted. You can only cancel common factors that multiply the entire numerator and denominator. To add or subtract fractions, find a common denominator by identifying the lowest common multiple of the individual denominators.

> **PAPER 1: NON-CALCULATOR STRATEGY**
>
> Look out for the difference of two squares identity ($a^2 - b^2$). Examiners frequently use this pattern to simplify large numerical products or fraction calculations without long division. For instance, evaluating $99^2 - 1$ is much faster when rewritten as $(99-1)(99+1) = 98 \times 100 = 9800$.

> **PAPER 2: CALCULATOR OPTIMIZATION**
>
> You can verify your algebraic factorizations by substituting a simple number (like $x = 2$) into both the original expression and your factorized result. If both evaluate to the exact same numerical value on your calculator, your factorization is correct.

Practice Problems & Step-by-Step Solutions

Problem 1: Simplifying Algebraic Fractions

Simplify the following expression into a single fraction: $[3 / (x - 2)] - [2 / (x + 3)]$

Solution:

Find the common denominator: $(x - 2)(x + 3)$.

Rewrite each fraction with the common denominator:

$= [3(x + 3) - 2(x - 2)] / [(x - 2)(x + 3)]$

Expand the terms in the numerator:

$= [3x + 9 - 2x + 4] / [(x - 2)(x + 3)] = [x + 13] / [(x - 2)(x + 3)]$

Problem 2: Advanced Factorization

Completely factorize the following expression: $4x^2 - 9y^2 + 2x - 3y$

Solution:

Identify the difference of two squares in the first two terms: $4x^2 - 9y^2 = (2x)^2 - (3y)^2 = (2x - 3y)(2x + 3y)$.

Rewrite the complete expression with this factorized part:

$= (2x - 3y)(2x + 3y) + 1(2x - 3y)$

Factor out the common binomial term $(2x - 3y)$:

$= (2x - 3y)(2x + 3y + 1)$

Week 3 Supplement: Analytical Mechanics of Geometric Lines

Coordinate geometry links algebraic equations to geometric shapes on a graph. Every straight line represents a constant rate of change, which is defined as its gradient (m). The standard equation of a straight line is:

$$y = mx + c$$

where m represents the gradient and c represents the y-intercept, the point where the line crosses the vertical axis.

Parallel and Perpendicular Lines

The relationship between two lines can be determined by comparing their gradients. Parallel lines run in the exact same direction and never intersect, meaning their gradients are equal ($m_1 = m_2$). Perpendicular lines intersect at a right angle ($90°$), and their gradients are negative reciprocals of each other, meaning $m_1 \times m_2 = -1$.

Comprehensive Practice Examples for Week 3

Extended Example 3.1: Finding the Perpendicular Bisector Equation

Find the equation of the perpendicular bisector of a line segment connecting point $A(2, 5)$ and point $B(6, 13)$.

Detailed Explanation:

Step 1: Find the midpoint of segment AB, which will be a point on our new line:

$Midpoint = ((2 + 6)/2, (5 + 13)/2) = (4, 9)$.

Step 2: Calculate the gradient of line AB (m_1):

$m_1 = (13 - 5) / (6 - 2) = 8 / 4 = 2$.

Step 3: Find the perpendicular gradient (m_2):

$m_2 = -1 / m_1 = -1 / 2$.

Step 4: Use the point-slope formula with the midpoint $(4, 9)$ and the perpendicular gradient:

$y - 9 = (-1/2)(x - 4) \implies y - 9 = (-1/2)x + 2 \implies y = (-1/2)x + 11$.

Week 3: Linear Equations, Graphs, & Coordinate Geometry

Syllabus Coverage & Core Concepts

This week focuses on linear math. Topics include solving multi-step linear equations, simultaneous linear systems, finding gradients, line lengths, midpoints, and working with parallel and perpendicular lines using the standard equation $y = mx + c$.

Comprehensive Theoretical Explanations

A straight line represents a constant rate of change. The gradient (m) measures this steepness and is calculated from any two points (x_1, y_1) and (x_2, y_2):

$$m = (y_2 - y_1) / (x_2 - x_1)$$

The distance between these two points is derived from Pythagoras' Theorem:

$$\text{Distance} = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

Geometric relationships between different lines depend entirely on their gradients:

- If two lines are parallel, their gradients are equal: $m_1 = m_2$.
- If two lines are perpendicular, their gradients are negative reciprocals: $m_1 \times m_2 = -1$.

> **PAPER 1: NON-CALCULATOR STRATEGY**
>
> When solving simultaneous equations using elimination, always choose to eliminate the variable with opposite signs. This allows you to add the equations together, which minimizes mistakes with negative numbers.

> **PAPER 2: CALCULATOR OPTIMIZATION**
>
> Many modern scientific calculators can solve simultaneous equations directly. Use this built-in feature to verify your answers, but make sure to show all your manual working steps on the exam paper to secure full marks.

Practice Problems & Step-by-Step Solutions

Problem 1: Perpendicular Line Equations

Find the equation of the line that passes through the point $(4, -1)$ and is perpendicular to the line with equation $2y - 3x = 6$.

Solution:

First, rearrange the given line equation into the standard $y = mx + c$ form:

$2y = 3x + 6 \implies y = (3/2)x + 3$. The gradient of this line is $m_1 = 3/2$.

Calculate the perpendicular gradient (m_2): $m_2 = -1 / (3/2) = -2/3$.

Use the point-slope formula with the point $(4, -1)$:

$y - y_1 = m_2(x - x_1) \implies y - (-1) = (-2/3)(x - 4)$

$y + 1 = (-2/3)x + 8/3 \implies y = (-2/3)x + 5/3$

Multiply by 3 to express in standard form: $3y + 2x = 5$.

Problem 2: Simultaneous Equations with Fractions

Solve the following system of linear equations: $2x + 3y = 11$ and $(x / 2) - y = -1$

Solution:

Multiply the second equation by 2 to clear the fraction: $x - 2y = -2$. Rearrange to find x: $x = 2y - 2$.

Substitute this expression for x into the first equation:

$2(2y - 2) + 3y = 11 \implies 4y - 4 + 3y = 11$

$7y - 4 = 11 \implies 7y = 15 \implies y = 15/7$

Substitute y back to find x: $x = 2(15/7) - 2 = 30/7 - 14/7 = 16/7$.

Week 4 Supplement: Deep Analytical Study of Functions and Graphs

Functions describe how an output variable changes in response to an input variable. Plotting these relationships visually allows us to study their geometric properties. While linear functions form straight lines, non-linear functions create distinct curves depending on their highest exponent power.

The Turning Point of a Parabola

A quadratic function forms a symmetrical curve called a parabola. The vertex, or turning point, is the absolute lowest point on a U-shaped curve (minimum) or the highest point on an n-shaped curve (maximum). Completing the square is an effective algebraic technique to find the exact coordinates of this vertex without graphing.

Comprehensive Practice Examples for Week 4

Extended Example 4.1: Vertex Derivation via Completing the Square

Express the quadratic function $y = 2x^2 - 12x + 11$ in the form $y = a(x - h)^2 + k$, and state the coordinates of its turning point.

Detailed Explanation:

First, factor out the coefficient of x^2 from the first two terms:

$y = 2(x^2 - 6x) + 11$

Take half of the linear x-coefficient ($-6 / 2 = -3$), square it ($(-3)^2 = 9$), and then add and subtract it inside the parenthesis:

$y = 2(x^2 - 6x + 9 - 9) + 11$

Group the perfect square trinomial terms together:

$y = 2((x - 3)^2 - 9) + 11$

Distribute the outer multiplier back to remove the outer brackets:

$y = 2(x - 3)^2 - 18 + 11 \implies y = 2(x - 3)^2 - 7$.

This reveals that the vertex is a minimum turning point located at the coordinates $(3, -7)$.

Week 4: Quadratic Equations, Functions, & Non-Linear Graphs

Syllabus Coverage & Core Concepts

This block focuses on non-linear equations. Topics include solving quadratic equations using factoring, completing the square, and the quadratic formula, as well as plotting and reading quadratic, cubic, and reciprocal graphs.

Comprehensive Theoretical Explanations

Quadratic functions have a distinct symmetrical shape called a parabola. The turning point, or vertex, represents the maximum or minimum value of the function. This turning point can be found by completing the square to rewrite the expression in the form $y = a(x - h)^2 + k$, where the vertex is located at the coordinates (h, k).

Cubic curves ($y = ax^3 + bx^2 + cx + d$) display an S-shaped inflected profile, while reciprocal models ($y = a / x$) produce hyperbolic branches that approach the coordinate axes but never touch them. These boundaries are known as asymptotes.

> **PAPER 1: NON-CALCULATOR STRATEGY**
>
> When drawing graphs, remember that the constant term in the equation always tells you the y-intercept. For example, the curve $y = 2x^2 - 4x - 6$ must cross the vertical y-axis at exactly $(0, -6)$. Use this point to check that your calculated points are accurate.

> **PAPER 2: CALCULATOR OPTIMIZATION**
>
> When completing a table of values for a curve, use parenthesis around negative numbers before squaring them on your calculator. For example, calculate $(-3)^2 = 9$. Typing -3^2 without parenthesis will output -9, which is a very common error.

Practice Problems & Step-by-Step Solutions

Problem 1: Solving via the Quadratic Formula

Solve the equation $3x^2 + 7x - 5 = 0$, giving your answers to two decimal places.

Solution:

Identify the coefficients for the formula: $a = 3, b = 7, c = -5$.

Substitute these values into the quadratic formula:

$$x = [-7 \pm \sqrt{(7^2 - 4(3)(-5))}] / (2 \times 3)$$

$$x = [-7 \pm \sqrt{(49 + 60)}] / 6 = [-7 \pm \sqrt{109}] / 6$$

Calculate the two potential values for x:

Case 1: $x = (-7 + 10.4403) / 6 = 0.573 \implies 0.57$

Case 2: $x = (-7 - 10.4403) / 6 = -2.906 \implies -2.91$

Problem 2: Intersecting Linear and Quadratic Graphs

Find the coordinates where the straight line $y = 2x + 1$ intersects the quadratic curve $y = x^2 - 2x - 4$.

Solution:

Set the two equations equal to each other to find their intersection points:

$$x^2 - 2x - 4 = 2x + 1$$

Move all terms to one side to set the quadratic equation to zero:

$$x^2 - 4x - 5 = 0$$

Factorize the quadratic expression: $(x - 5)(x + 1) = 0$. This gives $x = 5$ or $x = -1$.

Substitute these x-values back into the linear equation to find the corresponding y-coordinates:

For $x = 5 \implies y = 2(5) + 1 = 11$. First point: $(5, 11)$.

For $x = -1 \implies y = 2(-1) + 1 = -1$. Second point: $(-1, -1)$.

Week 5 Supplement: Linear Inequalities and Regional Constraints

Inequalities describe ranges of possible values rather than single, fixed solutions. When working with inequalities in two variables, we can represent them graphically as regions on a coordinate grid. Shading these regions allows us to solve optimization problems through linear programming.

Defining Boundary Lines

The boundary line of an inequality is drawn using the standard equation of a straight line. We use a solid line for inequalities that include the boundary ($\leq$ or $\geq$), meaning points on the line are part of the solution set. We use a dashed line for strict inequalities ($<$ or $>$) to show that points on the boundary are excluded.

Comprehensive Practice Examples for Week 5

Extended Example 5.1: Finding the Unshaded Region Boundaries

An unshaded region R on a graph is bounded by three lines: the y-axis, the line $y = 2$, and the line $x + y = 5$. Write down the three inequalities that define this region.

Detailed Explanation:

1. The region lies to the right of the vertical y-axis ($x = 0$), which gives the inequality: $x \geq 0$.

2. The region lies above the horizontal line $y = 2$, which gives the inequality: $y \geq 2$.

3. The region lies below and to the left of the slanted line $x + y = 5$. Testing a point inside the region confirms the inequality: $x + y \leq 5$.

Week 5: Inequalities, Linear Programming, & Sequences

Syllabus Coverage & Core Concepts

This week focuses on inequalities and patterns. Topics include solving linear inequalities, shading regions on a graph to define sets of inequalities, and identifying formulas for arithmetic, quadratic, and geometric sequences.

Comprehensive Theoretical Explanations

Solving linear inequalities follows the same basic rules as solving regular equations, with one crucial exception: whenever you multiply or divide both sides by a negative number, you must reverse the direction of the inequality sign.

In linear programming, target regions are defined on graphs by drawing boundary lines. Solid lines are used for inclusive inequalities ($\leq$ or $\geq$), while dashed lines indicate exclusive boundaries ($<$ or $>$).

For sequences, finding the general n-the term formula depends on the pattern of differences between terms:

- Arithmetic Sequences (constant first difference): $T_n = a + (n - 1)d$
- Quadratic Sequences (constant second difference): $T_n = an^2 + bn + c$

PAPER 1: NON-CALCULATOR STRATEGY

To determine which side of a boundary line to shade, pick a simple test point that is clearly not on the line, such as the origin $(0,0)$. Substitute these coordinates into your inequality. If it makes the statement true, the origin lies within the correct, desired region.

PAPER 2: CALCULATOR OPTIMIZATION

When working with sequences, you can use your calculator to check your n-th term formula. Plug in $n = 1, 2, 3$ to confirm that your formula generates the exact numbers in the original sequence list.

Practice Problems & Step-by-Step Solutions

Problem 1: Reversing Inequalities

Solve the inequality: $5 - 3(x - 2) \geq 20$

Solution:

Expand the brackets carefully, watching the negative signs: $5 - 3x + 6 \geq 20$.

Combine the constant numbers: $11 - 3x \geq 20$.

Subtract 11 from both sides: $-3x \geq 9$.

Divide by -3 and flip the inequality sign: $x \leq -3$.

Problem 2: Finding a Quadratic Sequence Formula

Find the n-th term formula for the following number sequence: $4, 9, 16, 25, 36, \ldots$

Solution:

Analyze the terms by rewriting them as squares: $4 = 2^2, 9 = 3^2, 16 = 4^2, 25 = 5^2$.

Notice that each term is the square of the position number plus one: $(n + 1)$.

Therefore, the general formula for the sequence is: $T_n = (n + 1)^2 = n^2 + 2n + 1$.

Week 6 Supplement: Advanced Structural Analysis of Regular Polygons

Geometry studies the properties and relationships of lines, angles, and shapes. Polygons are closed two-dimensional shapes made of straight line segments. In a regular polygon, all interior angles are equal and all side lengths are identical.

The Geometry of Exterior Angles

An exterior angle is formed by extending one of the sides of a polygon outwards. A key property of any convex polygon is that its exterior angles always sum to exactly ***360°***, no matter how many sides it has. This rule makes it easy to calculate individual angles and find the number of sides in a regular polygon.

Comprehensive Practice Examples for Week 6

Extended Example 6.1: Deducing Polygon Sides from Interior Angles

A regular polygon has an interior angle of ***162°***. Determine the total number of sides (***n***) of this polygon.

Detailed Explanation:

Step 1: Find the value of each individual exterior angle. Since interior and exterior angles on a straight line add up to ***180°***:

Exterior Angle = 180° - 162° = 18°.

Step 2: Use the total exterior angle sum rule (***360°***) to calculate the number of sides:

Number of Sides (n) = 360° / Exterior Angle = 360 / 18 = 20 ext{ sides}.

Week 6: Geometry — Angle Properties, Polygons, & Symmetry

Syllabus Coverage & Core Concepts

This block focuses on basic geometric principles. Topics include calculating angles across parallel lines, finding interior and exterior angles of polygons, identifying lines of symmetry, and determining rotational symmetry orders.

Comprehensive Theoretical Explanations

Angles follow fixed laws. When parallel lines are intersected by a straight transversal line, alternate angles are equal (forming a Z-shape), corresponding angles are equal (forming an F-shape), and interior angles on the same side add up to ***180°*** (forming a C-shape).

For any closed polygon with n sides, the sum of all interior angles is calculated using the formula:

$$\text{Sum} = (n - 2) \times 180°$$

In contrast, the exterior angles of any convex polygon always sum to exactly ***360°***, regardless of the number of sides. For a regular polygon where all sides and angles are equal, each individual exterior angle is simply:

$$\text{Exterior Angle} = 360° / n$$

PAPER 1: NON-CALCULATOR STRATEGY

Always state the geometric reason for every step in your angle calculations, even if the question doesn't explicitly ask for them. Examiners use these reasons to award partial credit if you make an arithmetic error in your final calculation.

PAPER 2: CALCULATOR OPTIMIZATION

When dealing with complex polygons, calculate the individual exterior angle first. It is often much faster to find interior angles by subtracting the exterior angle from ***180°*** than it is to use the full interior angle sum formula.

Practice Problems & Step-by-Step Solutions

Problem 1: Interior Angles of a Regular Polygon

Calculate the value of each interior angle in a regular octagon (an 8-sided shape).

Solution:

Method A: Using exterior angles.

The exterior angle of a regular octagon is: $360° / 8 = 45°$.

Since interior and exterior angles on a straight line add up to $180°$:

$\text{Interior Angle} = 180° - 45° = 135°$.

Problem 2: Solving Angles on Parallel Lines

In a parallel line configuration, an interior angle is expressed as $(2x + 10)°$ and its consecutive interior angle is $(3x - 5)°$. Find the value of x.

Solution:

Consecutive interior angles between parallel lines always sum to $180°$:

$(2x + 10) + (3x - 5) = 180$

Combine like terms: $5x + 5 = 180$

Subtract 5 from both sides: $5x = 175$

Divide by 5 to find x: $x = 35$.

Week 7 Supplement: Proofs and Logic in Circle Geometry

Circle theorems are geometric laws that define the relationships between angles, lines, and chords within a circle. Applying these theorems requires logical reasoning and geometric proofs.

The Alternate Segment Theorem

One of the more advanced circle theorems is the alternate segment theorem. It states that the angle formed between a tangent line and a chord through the point of contact is exactly equal to the angle subtended by that same chord in the opposite, alternate segment of the circle.

Comprehensive Practice Examples for Week 7

Extended Example 7.1: Multi-Theorem Angle Proof

A circle has a diameter AB. A point C lies on the circumference. A tangent line to the circle at point A intersects the line BC extended at point T. If angle $\angle ABC = 34°$, find angle $\angle ATC$.

Detailed Explanation:

1. Angle $\angle ACB$ is subtended by the diameter AB, so it is a right angle: $\angle ACB = 90°$.

2. In the right-angled triangle ΔABC, calculate the remaining angle $\angle BAC$: $\angle BAC = 90° - 34° = 56°$.

3. The line AT is a tangent, so it forms a right angle with the radius AB: $\angle BAT = 90°$.

4. In the large right-angled triangle ΔABT, calculate angle $\angle ATB$ (which is the same as $\angle ATC$): $\angle ATB = 90° - \angle ABC = 90° - 34° = 56°$.

Week 7: Circle Theorems & Geometric Proofs

Syllabus Coverage & Core Concepts

This block covers circle geometry. Topics include applying the eight major circle theorems, calculating angles inside inscribed triangles, and constructing logical geometric proofs for tangents and cyclic quadrilaterals.

Comprehensive Theoretical Explanations

Circle theorems are geometric rules that define relationships between angles, lines, and arcs within a circle. Recognizing these patterns is key to solving complex geometry problems.

Three foundational theorems are frequently tested together:

1. **Angle in a Semicircle:** Any angle subtended by a diameter at the circumference is always a right angle (***90°***).
2. **Tangent-Radius Theorem:** A tangent meets the radius of a circle at exactly ***90°*** at the point of contact.
3. **Tangents from an External Point:** Tangent lines drawn to a circle from the same external point are equal in length, creating a symmetrical kite shape.

PAPER 1: NON-CALCULATOR STRATEGY

Always draw a line from the center of the circle to any point where a tangent touches the circumference. This instantly creates right-angled triangles, allowing you to use basic angle rules or Pythagoras' Theorem to solve the problem.

PAPER 2: CALCULATOR OPTIMIZATION

When dealing with cyclic quadrilaterals, keep a checklist showing which pairs of opposite angles add up to ***180°***. This lets you quickly calculate missing angles across the circle using simple subtraction.

Practice Problems & Step-by-Step Solutions

Problem 1: Angles in a Cyclic Quadrilateral

Points ***A, B, C,*** and ***D*** lie on the circumference of a circle. If angle $\angle DAB = 78°$, find the value of the opposite interior angle $\angle BCD$.

Solution:

Because all four vertices touch the circle's circumference, shape ***ABCD*** is a cyclic quadrilateral.

Opposite angles in a cyclic quadrilateral always add up to $180°$:

$\angle BCD = 180° - \angle DAB$

$\angle BCD = 180° - 78° = 102°$.

Problem 2: Tangent and Isosceles Properties

A tangent line ***PT*** touches a circle at point ***T***. Point ***O*** is the center of the circle, and the line ***OP*** intersects the circle. If the radius is ***5 ext{ cm}*** and distance ***OP = 13 ext{ cm}***, find the length of the tangent segment ***PT***.

Solution:

According to the tangent-radius theorem, the angle $\angle OTP$ is exactly $90°$.

This means triangle ΔOTP is a right-angled triangle where ***OP*** is the hypotenuse.

Apply Pythagoras' Theorem: $OT^2 + PT^2 = OP^2$

$5^2 + PT^2 = 13^2$ ***\implies*** $25 + PT^2 = 169$

$PT^2 = 169 - 25 = 144$ ***\implies*** $PT = \sqrt{144} = 12$ ***ext{ cm}***.

Week 8 Supplement: Mathematical Ratios in Scale Models and Solids

When working with similar three-dimensional shapes, surface areas and volumes scale differently than linear lengths. Understanding these mathematical ratios is essential for solving engineering and modeling problems.

Surface Area and Volume Formulas

Calculating measurements for complex solids requires using standard geometric formulas:

- **Sphere:** $\text{Surface Area} = 4\pi r^2$, $\text{Volume} = (4/3)\pi r^3$
- **Cone:** $\text{Curved Area} = \pi rl$, $\text{Volume} = (1/3)\pi r^2 h$

Comprehensive Practice Examples for Week 8

Extended Example 8.1: Melting and Reshaping Solids

A solid metal sphere with a radius of *6 ext{ cm}* is melted down and recast into a solid cone with a base radius of *8 ext{ cm}*. Calculate the vertical height (h) of the cone.

Detailed Explanation:

Step 1: Calculate the volume of the sphere before it is melted:

$\text{Volume}_{sphere} = (4/3) \times \pi \times 6^3 = (4/3) \times \pi \times 216 = 288\pi \text{ ext\{ cm\}}^3$.

Step 2: Set the volume of the new cone equal to the volume of the sphere:

$\text{Volume}_{cone} = (1/3) \times \pi \times r^2 \times h \text{ \textbackslash implies } 288\pi = (1/3) \times \pi \times 8^2 \times h$

Step 3: Simplify the equation and solve for the height (h):

$288 = (64 / 3) \times h \text{ \textbackslash implies } h = (288 \times 3) / 64 = 864 / 64 = 13.5 \text{ ext\{ cm\}}$.

Week 8: Congruence, Similarity, & Mensuration of Solids

Syllabus Coverage & Core Concepts

This week focuses on scaling and measurement. Topics include proving triangle congruence, solving similar shapes, and calculating the surface area and volume of cylinders, cones, pyramids, and spheres.

Comprehensive Theoretical Explanations

Shapes are congruent if they are identical in size and shape. Triangles can be proven congruent using one of four criteria: Side-Side-Side (SSS), Side-Angle-Side (SAS), Angle-Side-Angle (ASA), or Right-angle-Hypotenuse-Side (RHS).

Similar shapes have the same matching angles, but their side lengths are scaled by a constant factor (k). When scaling similar areas and volumes, the relationships change predictably based on this scale factor:

$$\text{Area Ratio} = k^2 = (l_1 / l_2)^2$$

$$\text{Volume Ratio} = k^3 = (l_1 / l_2)^3$$

Mensuration involves using standard formulas to measure geometric solids. For example, the total surface area of a cone includes its circular base and its curved surface area: $A = \pi r^2 + \pi r l$, where l represents the slant height.

> **PAPER 1: NON-CALCULATOR STRATEGY**
>
> When working with similar shapes, always convert area or volume ratios back into a simple linear scale factor (k) by taking the square root or cube root first. Never use an area ratio to directly calculate a length.

> **PAPER 2: CALCULATOR OPTIMIZATION**
>
> Use the dedicated π button on your calculator instead of typing in 3.14 or $22/7$, unless the exam question explicitly tells you to use a specific value. This keeps your calculations accurate.

Practice Problems & Step-by-Step Solutions

Problem 1: Volume Scaling of Similar Containers

Two similar buckets have capacities of $2\ ext\{ liters\}$ and $16\ ext\{ liters\}$. If the smaller bucket has a height of $15\ ext\{ cm\}$, find the height of the larger bucket.

Solution:

Find the volume ratio: $V_{large} / V_{small} = 16 / 2 = 8$.

Since the volume ratio is k^3, find the linear scale factor (k) by taking the cube root:

$k = \sqrt[3]{8} = 2$.

Calculate the height of the larger bucket using the linear scale factor:

$Height_{large} = k \times Height_{small} = 2 \times 15 = 30\ ext\{ cm\}$.

Problem 2: Total Surface Area of a Cylinder

A solid cylinder has a radius of $3.5\ ext\{ cm\}$ and a height of $10\ ext\{ cm\}$. Calculate its total surface area, using $\pi = 22/7$.

Solution:

The formula for the total surface area of a solid cylinder is: $A = 2\pi r^2 + 2\pi rh$.

Substitute the given dimensions into the formula:

$A = 2 \times (22/7) \times (3.5)^2 + 2 \times (22/7) \times (3.5) \times 10$

Calculate the area of the two circular bases: $2 \times (22/7) \times 12.25 = 77\ ext\{ cm\}^2$.

Calculate the curved surface area: $2 \times (22/7) \times 3.5 \times 10 = 220\ ext\{ cm\}^2$.

Add both parts together for the total area: $A = 77 + 220 = 297\ ext\{ cm\}^2$.

Week 9 Supplement: Trigonometric Functions and Angle Relationships

Trigonometry defines the mathematical relationships between side lengths and angles in triangles. For right-angled triangles, these ratios are constant and depend entirely on the value of the acute angle.

Applications: Angles of Elevation and Depression

Angles of elevation and depression are used to measure heights and distances in real-world scenarios. An angle of elevation looks up from a horizontal line, while an angle of depression looks down. Because horizontal lines are parallel, these two angles are alternate angles and are equal to each other.

Comprehensive Practice Examples for Week 9

Extended Example 9.1: Multi-Step Right-Angled Triangle Problem

An observer at the top of a cliff $50 ext{ meters}$ high measures the angle of depression to a boat on the water as $24°$. After the boat moves closer to the cliff, the new angle of depression is $41°$. Calculate the distance the boat traveled.

Detailed Explanation:

Step 1: Find the initial horizontal distance (d_1) from the cliff using the first angle of depression ($24°$):

$tan(24°) = 50 / d_1 \implies d_1 = 50 / tan(24°) = 50 / 0.4452 = 112.31 ext{ m}$.

Step 2: Find the second horizontal distance (d_2) using the new angle of depression ($41°$):

$tan(41°) = 50 / d_2 \implies d_2 = 50 / tan(41°) = 50 / 0.8693 = 57.52 ext{ m}$.

Step 3: Calculate the distance traveled by subtracting the two horizontal distances:

$Distance\ Traveled = d_1 - d_2 = 112.31 - 57.52 = 54.79 ext{ m} \implies 54.8 ext{ m}$.

Week 9: Trigonometry — Right-Angled Triangles & 2D Applications

Syllabus Coverage & Core Concepts

This week focuses on basic trigonometry. Topics include using Pythagoras' Theorem, applying the trigonometric ratios (Sine, Cosine, Tangent), and solving word problems involving angles of elevation and depression.

Comprehensive Theoretical Explanations

Trigonometry defines the mathematical relationships between side lengths and angles in triangles. For any right-angled triangle, the lengths of the sides are linked to the acute angle θ by three standard ratios:

$$\sin \theta = Opp / Hyp$$

$$\cos \theta = Adj / Hyp$$

$$\tan \theta = Opp / Adj$$

Angles of elevation and depression are always measured from a flat, horizontal line of sight. An angle of elevation looks upward, while an angle of depression looks downward. Because horizontal lines are parallel, the angle of elevation from point A up to point B is always exactly equal to the angle of depression from point B down to point A.

> **PAPER 1: NON-CALCULATOR STRATEGY**
>
> Memorize the common right-angled triangle side combinations, known as Pythagorean triples, such as (3, 4, 5) and (5, 12, 13). Examiners frequently use these exact sets of numbers to keep non-calculator calculations clean and straightforward.

> **PAPER 2: CALCULATOR OPTIMIZATION**
>
> Always verify that your calculator is set to Degree mode (indicated by a "D" on the screen) before doing trigonometric calculations. If it is accidentally set to Radians ("R") or Gradians ("G"), your answers will be incorrect.

Practice Problems & Step-by-Step Solutions

Problem 1: Solving Sides with Tangent

A vertical flagpole casts a horizontal shadow that is ***12 ext{ meters}*** long on the ground. If the angle of elevation to the sun is ***38°***, calculate the height of the flagpole.

Solution:

The flagpole and the ground form a right angle. The shadow is adjacent to the angle, and the flagpole height is opposite.

Use the Tangent ratio: ***tan θ = Opp / Adj \implies tan(38°) = Height / 12***.

Rearrange the equation to solve for the height: ***Height = 12 × tan(38°)***.

Calculate the final value: ***Height = 12 × 0.7813 = 9.375 ext{ m} \implies 9.38 ext{ m}***.

Problem 2: Finding an Angle from Sides

A straight ladder measuring ***8 ext{ meters}*** long rests against a vertical wall. If the base of the ladder is ***3 ext{ meters}*** away from the wall, find the angle the ladder makes with the ground.

Solution:

The ladder acts as the hypotenuse (***8 ext{ m}***), and the distance along the ground is the adjacent side (***3 ext{ m}***).

Use the Cosine ratio: ***cos θ = Adj / Hyp \implies cos θ = 3 / 8***.

Convert to the decimal value: ***cos θ = 0.375***.

Use the inverse cosine function to find the angle: $\theta = \cos^{-1}(0.375)$.

Calculate the final angle: ***θ = 67.97° \implies 68.0°***.

Week 10 Supplement: Advanced Vector Navigation and Triangle Proofs

Advanced trigonometry allows us to solve problems for triangles that do not contain a right angle. By using the Sine and Cosine rules, we can find unknown side lengths and angles for any triangle shape.

The Sine Rule and the Ambiguous Case

When using the Sine Rule to find an angle given two sides and a non-enclosed angle (SSA), you may encounter the ambiguous case. Because the sine of an angle is equal to the sine of its supplementary angle ($\sin \theta = \sin(180° - \theta)$), there can be two distinct valid triangle shapes that fit the given measurements.

Comprehensive Practice Examples for Week 10

Extended Example 10.1: Navigation Using Bearings and the Cosine Rule

A ship leaves port P and sails 12 km on a bearing of $060°$ to position A. It then changes course and sails 15 km on a bearing of $130°$ to position B. Calculate the direct distance from port P to position B.

Detailed Explanation:

Step 1: Draw a diagram and calculate the interior angle at position A. The back bearing from A to P is $60° + 180° = 240°$.

The angle from the North line at A turning clockwise to line AB is $130°$. The interior angle $\angle PAB = 360° - (240° - 130°) = 110°$.

Step 2: This matches the Side-Angle-Side (SAS) pattern, so we apply the Cosine Rule to find distance PB:

$$PB^2 = PA^2 + AB^2 - 2(PA)(AB) \cos(\angle PAB)$$

$$PB^2 = 12^2 + 15^2 - 2(12)(15) \cos(110°)$$

$$PB^2 = 144 + 225 - 360 \times (-0.3420) \implies PB^2 = 369 + 123.12 = 492.12$$

Step 3: Take the square root to find the direct distance: $PB = \sqrt{492.12} = 22.18 \text{ km} \implies 22.2 \text{ km}$.

Week 10: Advanced Trigonometry — Oblique Triangles & Bearings

Syllabus Coverage & Core Concepts

This block covers non-right triangles. Topics include calculating triangle areas using sine, applying the Sine Rule and Cosine Rule, and solving navigation problems using three-figure bearings.

Comprehensive Theoretical Explanations

Non-right angled triangles cannot be solved with basic trigonometric ratios. Instead, we use advanced laws that work for any triangle shape. The choice of which rule to apply depends on the specific information given in the problem:

- **The Sine Rule:** Use this rule when you know a matching pair of an angle and its opposite side.

$$a / \sin A = b / \sin B = c / \sin C$$

- **The Cosine Rule:** Use this rule when you know two sides and the angle between them (SAS), or when you know all three sides (SSS).

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Bearings are specialized navigation angles used to show direction. Bearings must always follow three strict rules: they are measured from true North, they are measured turning clockwise, and they must always be written using three digits (for example, $045°$).

> **PAPER 1: NON-CALCULATOR STRATEGY**
>
> Remember that the sine of an obtuse angle is exactly the same as the sine of its acute supplement: $\sin(150°) = \sin(30°) = 0.5$. This identity is very useful for solving obtuse triangle questions without a calculator.

> **PAPER 2: CALCULATOR OPTIMIZATION**
>
> When using the Cosine Rule to find an angle, calculate the entire numerator and denominator separately before dividing. This prevents syntax errors and ensures your calculator follows the correct order of operations.

Practice Problems & Step-by-Step Solutions

Problem 1: Applying the Cosine Rule

In triangle ΔABC, side $AB = 7 ext{ cm}$, side $BC = 9 ext{ cm}$, and the enclosed angle $\∠ABC = 64°$. Calculate the length of the remaining side AC.

Solution:

This matches the Side-Angle-Side (SAS) pattern, so we apply the Cosine Rule:

$b^2 = a^2 + c^2 - 2ac \cos B$

Substitute the given side lengths and angle into the formula:

$AC^2 = 9^2 + 7^2 - 2(9)(7) \cos(64°)$

$AC^2 = 81 + 49 - 126 \times (0.43837)$

$AC^2 = 130 - 55.235 = 74.765$

Take the square root to find the final length: $AC = \sqrt{74.765} = 8.646 ext{ cm} \implies 8.65 ext{ cm}$.

Problem 2: Working with Back Bearings

The bearing of point B from point A is given as $115°$. Calculate the back bearing of point A when measured from point B.

Solution:

Draw parallel North lines at both point A and point B. The line connecting them creates interior angles.

Because the forward bearing is less than $180°$, find the back bearing by adding $180°$:

$Back\ Bearing = 115° + 180° = 295°$.

Week 11 Supplement: Advanced Statistical Histograms and Data Densities

Statistics organizes and analyzes datasets to reveal trends and patterns. For grouped continuous data, formatting the information correctly is vital to avoid distorting the results.

Histograms and Unequal Class Intervals

In a standard bar chart, the height of the bar represents the frequency. However, if the class widths are not equal, wider groups will look disproportionately large. To fix this, we use a histogram where the *area* of the bar represents the frequency, and the height is plotted as frequency density.

Comprehensive Practice Examples for Week 11

Extended Example 11.1: Constructing a Complete Frequency Density Table

A dataset tracking test completion times shows: $0 < t \leq 10$ minutes (Frequency: 12), $10 < t \leq 15$ minutes (Frequency: 15), and $15 < t \leq 30$ minutes (Frequency: 18). Calculate the frequency density for each interval.

Detailed Explanation:

Calculate the width and frequency density for each group using the formula: $FD = Frequency / Width$.

1. First Interval ($0 < t \leq 10$): Width = 10. $FD = 12 / 10 = 1.2$.

2. Second Interval ($10 < t \leq 15$): Width = 5. $FD = 15 / 5 = 3.0$.

3. Third Interval ($15 < t \leq 30$): Width = 15. $FD = 18 / 15 = 1.2$.

Week 11: Statistics — Data Representations & Central Tendency

Syllabus Coverage & Core Concepts

This week focuses on analyzing data. Topics include calculating Mean, Median, and Mode, constructing histograms with unequal class widths, and interpreting frequency density.

Comprehensive Theoretical Explanations

Statistical analysis summarizes raw data to reveal patterns. The three primary measures of central tendency describe the middle or typical value of a dataset in different ways:

- **Mean:** The arithmetic average, calculated by dividing the sum of all values by the total number of items.
- **Median:** The middle value when the data points are arranged in order from smallest to largest.
- **Mode:** The most frequently occurring value in the dataset.

When graphing grouped data where the class intervals are not equal, a standard bar chart can distort the data. Instead, we use a histogram where the *area* of the bar represents the frequency, not its height. The vertical axis must be plotted as Frequency Density:

$$\text{Frequency Density} = \text{Frequency} / \text{Class Width}$$

PAPER 1: NON-CALCULATOR STRATEGY

When finding the median from a grouped frequency table, do not guess. Construct a quick running total column (cumulative frequency) to find exactly which group contains the middle data point.

PAPER 2: CALCULATOR OPTIMIZATION

When calculating the mean of a grouped frequency table, use the formula $\text{Mean} = \Sigma fx / \Sigma f$. Multiply each frequency (f) by the exact midpoint (x) of its class interval, add them all up, and divide by the total frequency.

Practice Problems & Step-by-Step Solutions

Problem 1: Frequency Density Calculation

In a histogram, a class interval for weights is specified as $20 < w \leq 50$ and has an actual frequency count of ***45***. Find the frequency density height for this bar.

Solution:

First, find the width of the class interval: ***Class Width = 50 - 20 = 30***.

Apply the frequency density formula:

Frequency Density = Frequency / Class Width

Frequency Density = 45 / 30 = 1.5.

The bar on the histogram should be drawn with a height of 1.5 units on the vertical axis.

Problem 2: Grouped Mean Analysis

Calculate the estimated mean for the following frequency distribution: Distance $0 < d \leq 10$ (Frequency: 8) and Distance $10 < d \leq 30$ (Frequency: 12).

Solution:

Step 1: Identify the midpoints (x) for each class interval.

Midpoint 1 = $(0 + 10) / 2 = 5$

Midpoint 2 = $(10 + 30) / 2 = 20$

Step 2: Calculate the product of frequencies and midpoints ($f \times x$).

$\Sigma fx = (8 \times 5) + (12 \times 20) = 40 + 240 = 280$

Step 3: Divide by the total frequency count (Σf).

$\Sigma f = 8 + 12 = 20$

Estimated Mean $= \Sigma fx / \Sigma f = 280 / 20 = 14$.

Week 12 Supplement: Advanced Probability Theory and Sequential Sample Spaces

Probability measures the likelihood of events occurring. When analyzing multiple events that happen one after another, we use tree diagrams and Venn diagrams to map out all possible outcomes.

Conditional Probability

Conditional probability looks at how the likelihood of an event changes based on the outcome of a previous event. This is common in problems involving drawing items "without replacement," where the total number of available choices decreases with each turn.

Comprehensive Practice Examples for Week 12

Extended Example 12.1: Multi-Branch Venn and Probability Intersection Analysis

In a group of ***30 students***, ***18 study Additional Mathematics***, ***15 study Physics***, and ***7 study both subjects***. Find the probability that a randomly selected student studies neither of these subjects.

Detailed Explanation:

Step 1: Map the data using a Venn diagram with two overlapping sets, Math (M) and Physics (P).

Place the number of students who take both subjects in the overlapping intersection: $n(M \cap P) = 7$.

Step 2: Calculate the number of students who take Math only and Physics only:

Math only = $18 - 7 = 11$

Physics only = $15 - 7 = 8$

Step 3: Add these groups together to find the total number of students taking either subject:

Total = 11 + 7 + 8 = 26 students.

Step 4: Subtract this from the total group size to find the students who take neither subject: ***30 - 26 = 4 students***.

Therefore, the probability is: $P(\text{Neither}) = 4 / 30 = 2 / 15$.

Week 12: Probability — Dispersion Trends, Trees, & Venn Models

Syllabus Coverage & Core Concepts

This final block covers dispersion and probability. Topics include reading cumulative frequency curves, finding the Interquartile Range (IQR), and solving probability problems using tree diagrams and Venn diagrams.

Comprehensive Theoretical Explanations

Dispersion measures how spread out the values are in a dataset. While the range only looks at the absolute extremes, the Interquartile Range (IQR) measures the spread of the middle 50% of the data, making it a more reliable measure that ignores unusual outliers:

$$\textit{Interquartile Range (IQR)} = \textit{Upper Quartile } (Q_3) - \textit{Lower Quartile } (Q_1)$$

Probability measures the likelihood of an event occurring, on a scale from 0 (impossible) to 1 (certain). For compound events that happen one after another, we map the options using probability tree diagrams.

Follow two fundamental rules when calculating paths on a tree diagram: multiply probabilities along a continuous branch path to find the likelihood of combined independent outcomes, and add the resulting probabilities across separate branch paths.

> **PAPER 1: NON-CALCULATOR STRATEGY**
>
> When a question states that items are drawn "without replacement," remember to decrease both the numerator and the denominator for the second set of branches on your tree diagram, as the total pool of items has changed.

> **PAPER 2: CALCULATOR OPTIMIZATION**
>
> When reading quartiles from a cumulative frequency curve, write down the exact operational values on your graph lines (such as 25% for Q1 and 75% for Q3) to show the examiner your method clearly.

Practice Problems & Step-by-Step Solutions

Problem 1: Interquartile Range from Quartiles

A cumulative frequency graph of exam scores shows a Lower Quartile (Q_1) of ***42 marks*** and an Upper Quartile (Q_3) of ***71 marks***. Find the Interquartile Range.

Solution:

The Interquartile Range measures the difference between the upper and lower quartiles:

$IQR = Q_3 - Q_1$

$IQR = 71 - 42 = 29$ ext{ marks}.

Problem 2: Dependent Probability Trees

A bag contains ***5 red marbles*** and ***3 blue marbles***. Two marbles are drawn at random one after another without replacement. Find the probability that both marbles are red.

Solution:

Calculate the probability of drawing a red marble first: $P(Red_1) = 5 / 8$.

Since the first marble is not replaced, update the count: 4 red and 3 blue marbles remain (7 total).

Calculate the probability of drawing a red marble second: $P(Red_2) = 4 / 7$.

Multiply the probabilities along the branch path to find the combined outcome:

$P(Both\ Red) = P(Red_1) \times P(Red_2) = (5 / 8) \times (4 / 7)$

$P(Both\ Red) = 20 / 56 = 5 / 14$.